1978 KANSAS WINTER WHEAT YIELD ESTIMATION AND MODELING

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ABSTRACT

The report discusses a research effort directed toward improved methods of forecasting and estimating final harvested winter wheat yield at the field level. Reliable forecasts of the weight per head component are made in four fields in Ellsworth County, Kansas using a constrained logistic growth model. Yield estimation is done in two ways. Yield is estimated directly from grain weights obtained just before harvest and also by using a logistic growth model. The two yield estimates compare well. Data are obtained at the elevator to provide actual production. All yield estimates are higher than the derived elevator yields. A relationship between flowering date and grain weight is identified.

Key words: Yield; logistic growth model; flowering date; maturity stage; sampling bias.

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Introduction

Research has been conducted for the past several years in modeling the dry head weight for wheat. The purpose of this effort has been to develop a model which can make early season forecasts of yield per acre based on current season data as a supplement to the regular forecasting program. The regular objective yield forecast puts current year plant data into models developed from previous years. While performing satisfactorily in many years, models which have been developed from historic sources may falter in atypical years. Within-year growth modeling research has been conducted with the idea of providing supplemental information or sensitivity in unusual years.

The emphasis of the 1978 Study differed from past research in that harvested grain yield forecasts and estimates were to be made at the field level. By contrast, the 1977 Study (Larsen, 1978) was conducted in such a way that inferences could be made at the state level. The change to modeling at the field level in the current study as opposed to fitting a single model to data from many fields was done to gain better insight into factors influencing the growth model without having to contend with between field variability caused by variety, weather or other factors. While it is conceivable that field level forecasts could be aggregated to provide large area inference, it is recognized that the relatively large sample size in each field would likely be cost prohibitive in an operational program.

The primary objectives of the 1978 Study were to forecast mean harvested yield at the field level and to obtain estimated and elevator net yield for comparison. The data were to provide sufficient information to evaluate the forecasting capability of the growth model and the methods used to obtain forecasted and estimated net yield.

Sample Design and Data Collection

The sample design of the 1978 Study directly deals with several problems encountered in the 1977 Study. During the 1977 growing season, there was as much as a 2-week gap between the earliest flowering stalks and those which flowered last within individual fields. It was noted that aggregation of stalk data on a fixed calendar date basis corresponding to sampling visits might average over widely differing lengths of time since flowering. This could obscure any real time-growth relationship since almost all of the mature grain weight is present within 25 days after flowering. Based on this experience, a new sample design was developed for the 1978 Study.

Four rectangular fields were chosen in Ellsworth County in north central Kansas. They were numbered for consistent identification throughout the study. Field 1 was 30 acres, field 2 was 11 acres, field 3 was 30 acres, and field 4 was 46 acres.

Twenty-four random plot locations were chosen in each field. Each plot consisted of one 5-foot section of row. In the event that a random plot location fell in an area where rows were not distinguishable, the plot was defined to be 6 inches wide. Otherwise, the row width determined the width of each plot.

During the last week of April, all live stalks were counted within each plot so that an estimate of the stalk population could be made for the field. The total number of stalks in each plot was divided by 100 and rounded to the nearest integer. This number was then used to identify which stalks were to be tagged. Within each plot, 100 stalks were tagged and given a number. For example, if there were 487 total stalks in the plot, every fifth stalk would be tagged. This was done so that the tagged stalks would always correspond closely to the 5-foot area and not be bunched together at the front of the plot or extend any great distance past the plot. The position of the last tagged stalk provided a rough check on the stalk count. If the last stalk was very far from the end of the plot or on the wrong side of the end as determined by the direction of rounding to obtain the integer, the stalks were recounted.

After all the plots were laid out and stalks tagged, the next event was the observation of flowering. Flowering is evidence that a spikelet is fertile and will produce at least one kernel if external factors do not intervene. During the last two weeks of May, each tagged stalk was observed for the occurrence of flowering by making visits on Monday, Wednesday and Friday in fields 1 and 2 and on Tuesday, Thursday and Saturday in fields 3 and 4. The flowering date was estimated by taking the average of the date of the last unflowered visit and the first flowered visit. This provided estimates of the flowering date for each tagged stalk which flowered with a maximum error of 1.5 days.

As flowering progressed, the tagged stalks were grouped into so-called maturity stages. The first maturity stage was defined on the first visit in which at least seven stalks had flowered and it included all stalks flowered by that visit. Subsequent maturity stages were defined on each visit in which at least seven additional heads were observed to have flowered and contained all flowered stalks not previously assigned to a maturity stage. The last maturity stage was somewhat of a problem because not all stalks flower and there may or may not have been seven additional flowered stalks available. The last maturity stage therefore contained one or more stalks. Thus, with the possible exception of the first and last maturity stages in a plot, each maturity stage consisted of stalks with a common flowering date. Based on the 1977 growing season, the flowering period was expected to span about three weeks within a field with most of the flowering concentrated in the second week. This implied an average of about seven maturity stages in each plot. The 1978 season proved to be somewhat different in Ellsworth County. A long cool spring caused an unusually rapid flowering period when the temperatures suddenly warmed up. The flowering period spanned a 2-week interval in each of the four fields with most of the flowering during a 7-day period. However, on the plot level where the maturity stages were being defined, most flowering was observed on a single visit with the remaining stalks either flowering within a 1-week period or spanning 2 weeks so thinly that the first and last maturity stages each included multiple flowering dates. The result was that most of the 96 total plots had only three maturity stages. Some had two maturity stages and a few had four but there were no plots with four maturity stages all containing seven or more stalks.

After a maturity stage had been defined, one head was clipped each week until harvest. Clips were made on Friday or Saturday depending on the field. If a maturity stage was defined on a Friday or Saturday, a head was clipped that same day. Otherwise, clipping began the following Friday or Saturday. Harvest occurred between 5 and 6 weeks after initial flowering in all four fields. There were sufficient heads to clip up until harvest in all but some of the last maturity stages to be defined within a plot.

The heads to be clipped on a particular weekly clip day were determined using a method which ensured random selection. As flowering was being observed, the end of the tag of each flowered stalk was clipped and placed in a plain letter-sized envelope. The stalk tags were printed with two numbers so that the end could be clipped and stalk identification still retained. This procedure prevented the same stalk from somehow being included in more than one maturity stage and made the observance of flowering on subsequent visits easier by limiting the set of currently flowered stalks to those with whole tags. When a maturity stage was defined, there was a corresponding envelope containing all the clipped tag ends with stalk numbers for all stalks in the maturity stage. The stalks to be clipped in the maturity stage were determined by drawing tag ends without replacement and allocating the corresponding stalk numbers to seven potential clip days in the order in which they were drawn. For those maturity stages with fewer than seven stalks, stalk numbers were allocated until they ran out. For these maturity stages no heads were sampled as harvest approached causing some problems in the estimation of final grain yield. This is discussed in a later section. Alternatively, the stalks could have been randomly allocated over all the seven potential clip days but this was not done because, from a forecasting standpoint, it is preferable to have the observations in the early part of the grain development.

Each clipped head was placed in an airtight plastic tube, uniquely identified and mailed to a laboratory located in the State Statistical Office in Topeka, Kansas. Wet and dry head weights were determined to the nearest centigram for each sampled head. The drying process was 46 hours long at approximately 150°F. This temperature was used to ensure that there would be no destruction of dry matter in the immature heads.

On the last clip day before harvest, an extra head associated with each maturity stage was clipped. The extra heads usually came from untagged stalks right next to the regularly sampled heads. Harvesting occurred no more than two days after the last clips were made. Lab procedures for the heads sampled on the last visit were somewhat different. On the regular sample, wet and dry head weights were determined as described before. However, after the dry head weight was determined, the fertile spikelets were counted and the kernels were extracted and counted. The kernels were then returned to an oven for one hour at a temperature of approximately $270^{\circ}F$. This was done to drive out any moisture that may have been absorbed during the kernel extraction process. After this drying period, a dry kernel weight per head was determined. For each of the extra heads a fertile spikelet count and kernel count was made. A wet and dry kernel weight was then determined. The drying process for the mature kernels was 16 hours at approximately $270^{\circ}F$.

The purpose of obtaining the extra heads was to calculate a dry kernel to dry head weight ratio which when multiplied by the forecasted dry head weight at maturity would provide a forecast of dry kernel weight per head at maturity. In the 1977 Study, dry kernel weights were obtained for the extra heads only. The variability in the dry head to dry kernel ratio from adjacent heads was so large that relying on a mean ratio to adjust to mean dry kernel weight per head at harvest was not reliable. The 1978 Study directly addresses this problem by obtaining dry kernel weight on the same head for which dry head weight was determined. It is then possible to assess the affect of using a mean ratio from adjacent heads.

When harvest arrived, the cooperating farm operators in the study harvested the fields separately. The grain was taken to the elevator by the farm operator and weight and moisture content were determined. The field enumerators measured the size of the rectangular fields by pacing around the perimeter and applying an average length of pace to convert to acres. The farm operators were also asked to provide acreage estimates. The farmer estimates were higher than the calculated estimates made from counting paces. The differences ranged from 4 percent in field 3 to 13 percent in field 2.

Shortly after a field was harvested, eight post-harvest plots were laid out. The procedures were the same as in the Wheat Objective Yield Program. An average harvest loss per acre was estimated for each field.

More detailed information on data collection procedures can be found in the Enumerator's Manual and Laboratory Manual which are referenced in the back of this report.

Logistic Growth Model

The logistic growth model has been used in previous research to describe the time-growth relationship during grain filling. Several earlier reports which make use of this model are listed in the reference section. In the case of winter wheat, the growth model has been used to describe the relationship between dry head weight and time since flowering. The basic form of the growth model is as follows:

(1) $y_i = \dfrac{\alpha}{1 + \beta\rho^t} + \varepsilon_i$ where $i = 1, 2, \ldots, n$

$\alpha>0,\ \beta>0,\ 0<\rho<1$

y_i = dependent growth variable

t_i = independent time variable

ε_i = error term

Least squares theory was used to estimate the parameters α, β, and ρ. This requires the following assumptions about the nature of the model.

(2) $E(\varepsilon_i) = 0$ for all i

(3) $Var(\varepsilon_i) = E(\varepsilon_i^2) = \sigma^2$ for all i

(4) $Cov(\varepsilon_i, \varepsilon_j) = E(\varepsilon_i \varepsilon_j) = 0$ for all $i \neq j$

The parameter which we are most interested in estimating is the asymptote, α (see Diagram 1). The asymptote is the average amount of dry matter per head which has been accumulated when the time variable is very large. Depending on how fast the growth model converges, a near asymptotic value may not be reached until sometime after harvest. Therefore, the growth model can be truncated to provide an estimate of the mean dry head weight at harvest. To obtain a forecast of harvested yield, the mean dry weight per head is adjusted for threshing and moisture to the standard moisture grain weight per head. The stalk counts that are made in the 5-foot plots are adjusted by the proportion which produce heads and expanded to an average number of heads per acre at harvest. Multiplying the standard moisture grain weight per head times the average number of heads per acre at harvest produces a forecast of biological yield per acre. Net yield is obtained by subtracting the mean harvest loss calculated from the harvest loss plots.

There is a relationship between the stalk population in a plot and the mean dry head weight derived from a plot. The stalk population and dry head weight tend to be negatively correlated so that denser stands produce less dry matter per head than thinner stands. To properly reflect this relationship, individual observations need to be weighted by the corresponding stalk population per unit area. To obtain an estimate of the flowered stalk population at the plot level, the original stalk counts were multiplied by the proportion of the 100 tagged stalks which actually flowered. The estimated number of flowered stalks in the plot was partitioned to the maturity stages based on the number of flowered stalks in each maturity stage. An arbitrary unit of one square foot was used to keep the magnitude of the stalk population estimates relatively small. To offset differences in sampling intensity, the estimate of the number of flowered stalks per square foot for each maturity stage was divided by the number of stalks sampled in the maturity stage. Therefore, the

Logistic Growth Model

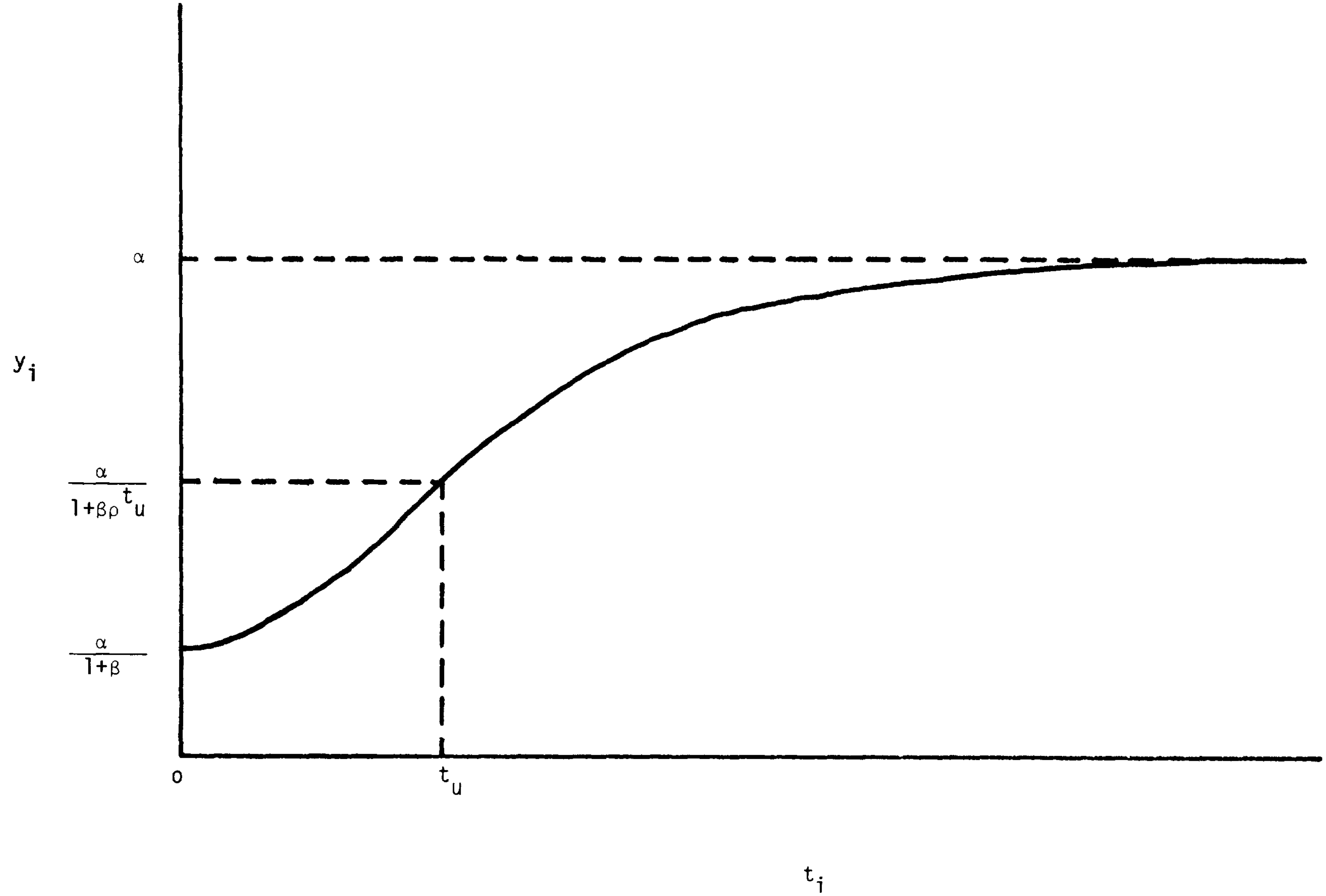


Diagram 1

individual observations were weighted by the estimated number of flowered stalks per square foot for each maturity stage divided by the number of stalks sampled in the maturity stage.

An alternative form of the logistic growth model was used to adjust for a violation of the assumption in (3) and is as follows:

$$(5) \quad w_i y_i = w_i \left[\frac{\alpha}{1 + \beta\rho^t} + \varepsilon_i \right] \quad i = 1, 2, \ldots, n$$

In practice, the variance of the dry head weights is not constant over the range of time. Therefore, an adjustment is needed to avoid making errors in the least squares estimation procedure. This violation of the assumption of equal variance over time is commonly referred to as heteroscedasticity and has been dealt with in some detail in an earlier report by Larsen (1978). The presence of heteroscedasticity is evidenced by a significant correlation between the absolute value of the residuals and time. Several adjustment techniques have been used in the past to produce residuals whose absolute values appear to have a random relationship with time. One of these is to describe the unknown, continuous and increasing relationship between the standard deviation of dry head weight and time by a linear function. The form of this function is described in Larsen (1978). In the 1978 data set where the occurrence of flowering was observed every two days, a step function was thought to adequately represent this unknown continuous relationship. To do this, a dry head weight standard deviation was calculated for intervals of time which are approximately two days wide. The value for w_i in (5) was $\hat{\sigma}/\hat{\sigma}_t$ where $\hat{\sigma}$ is the square root of the mean square error from an initial fit of the model in which no heteroscedasticity adjustment had been made. The $\hat{\sigma}_t$ is the calculated standard deviation of the mean dry head weight for various time intervals. The justification for using this ratio to adjust for heteroscedasticity is contained in the earlier report by Larsen (1978). In this report the heteroscedasticity adjustment was only used where noted.

Estimation of Mean Dry Head Weight

An initial fit of the logistic growth model in (1) was made for each field. The adjusted model in (5) was fit using the resultant mean square errors. Results are presented in Table 1 for both the unadjusted and adjusted models.

A few of the column headings in Table 1 need explanation. R^2 is the square of the multiple correlation coefficient obtained from the regression fit and is an indicator of how much of the total variability is being accounted for by the model. The relative standard error (RSE) is the standard error of $\hat{\alpha}$ divided by $\hat{\alpha}$. This is expressed as a percentage. R_p is the Pearson correlation between the absolute value of the residuals and time. The statistical significance of this correlation is an indicator of the degree of heteroscedastic disturbance in the model. A significance probability is given for each

Table 1.--Growth model fit at field level

Field	Hetero. Adjust.	Obs.	MSE	R^2	$\hat{\alpha}$ (g)	RSE (%)	R_p	Prob>$\|R_p\|$
1	No	306	.027	.944	.747	3.2	.405	.0001
	Yes	306	.026	.945	.754	3.6	-.015	.7824
2	No	381	.051	.933	.959	3.7	.401	.0001
	Yes	381	.052	.940	.948	3.0	-.015	.7711
3	No	273	.034	.937	.948	6.8	.421	.0001
	Yes	273	.033	.939	.946	6.0	.068	.2652
3F	No	145	.031	.939	1.014	11.0	.425	.0001
	Yes	145	.031	.943	.974	8.2	.073	.3836
3NF	No	128	.032	.940	.800	6.0	.487	.0001
	Yes	128	.031	.944	.848	8.0	.063	.4825
4	No	326	.048	.916	.923	5.7	.398	.0001
	Yes	326	.047	.926	1.022	7.1	-.106	.0566

Table 2.--Mean dry head weight at harvest

Field	t (days)	$\hat{y}$ (unadj.) (g)	$\hat{y}$ (adj.) (g)
1	36	.729	.731
2	36	.908	.906
3	35	.858	.856
3F	35	.893	.878
3NF	35	.766	.782
4	37	.866	.891

Pearson correlation coefficient. This is the probability that an equal or greater correlation (in absolute terms) than the one calculated would have arisen from another random sample given that the residuals and time are truly uncorrelated. For example, at a significance level of .05, we would accept the hypothesis of no correlation if the significance probability is greater than .05. We would fail to accept for probabilities less than or equal to .05 and conclude that the residuals and time are not uncorrelated.

Field 3 was subdivided into two parts: fallow and no fallow. Roughly half of field 3 was summer fallowed and this caused a difference in the stand and grain production. It might be advantageous to model the fallow separately if the fitted growth curves differed significantly in shape. In this case, the fitted curve for field 3 as a whole was nearly the same shape as those for the subdivisions. The only difference was the asymptote corresponding to mean dry head weight at maturity. So long as the fitted curves are similar in shape, the curve fitted using all the data from field 3 will do a good job of estimating α at the weighted average of the $\hat{\alpha}$'s from the fallow and no fallow fitted curves.

Several observations can be made from Table 1. The R^2 values indicate that the nonlinear regression is accounting for most of the variation. The $\hat{\alpha}$ values do not always increase when the heteroscedasticity adjustment is used. (Increases consistently occurred in the 1977 Study.) The heteroscedasticity adjustment is successful in creating uncorrelated residuals.

It was expected that the RSE's would tend to be lower when fitting the model at the field level rather than over many fields. The reason for this is that many factors influencing yield such as weather, soil and variety would be less variable within a field than over many fields. The RSE from the 1977 Study was approximately 4 percent. It can be seen in Table 1 that the RSE for fields 1 and 2 is slightly less than 4 percent while the RSE in the other fields is somewhat more. An explanation for this apparently lower than expected precision is that the model in the 1977 Study was fit to 366 aggregated observations. Using aggregated data points rather than "raw" data, as was done in 1978, tends to reduce the variability present in the population as the model sees it. Taking this into consideration, the RSE's in Table 1 are consistent with expectations.

The magnitude of the change in $\hat{\alpha}$ when the heteroscedasticity adjustment is used is somewhat misleading. The fitted growth curves should be truncated at the time of harvest if the estimated mean dry head weight ($\hat{y}$) at harvest is appreciably less than $\hat{\alpha}$. This was done and the information is presented in Table 2. The $\hat{y}$ values for the unadjusted model are less than the corresponding $\hat{\alpha}$ values. A paired t-test with five degrees of freedom showed a significant difference at $\alpha = .02$. The $\hat{y}$ values for the adjusted model decreased such that the differences between $\hat{y}$ values for the unadjusted and adjusted fits are less. A paired t-test with five degrees of freedom on the absolute differences between unadjusted and adjusted for $\hat{\alpha}$ and $\hat{y}$ showed a significant difference at $\alpha = .10$. This suggests that the reason the adjustment can cause a large increase in $\hat{\alpha}$ may be that the shape of the curve may change such that it takes longer for convergence.

Modeling Forecasts of Mean Dry Head Weight

To be of any real value, the growth model must be able to forecast the final mean dry head weight prior to harvest since estimation at harvest can be better accomplished by crop cutting. To be of benefit to the regular objective yield forecasting program as it currently exists, the growth model has to produce a June 1 forecast with reasonable accuracy. This has always been a difficult objective due to the relatively short grain filling period and the close proximity of the flowering date to June 1. In 1978, most of the flowering occurred during the third week of May. June 3 was the first clip date after all maturity stages had been defined. At this time, there were at most two weeks of data available. Growth model forecasts were made for June 3, June 10 and June 17 and compared to the final growth model estimated mean dry head weight obtained from data collected up through June 24 which was just prior to harvest. The forecasts were also compared to the mean dry head weight calculated directly from heads sampled on the last visit.

It was suggested by Rockwell (1978) that better forecasts of mean dry kernel weight per plant could be obtained for corn by using prior information to place constraints on the estimated value of ρ. An investigation was made to see if parameter constraints would improve forecasts for wheat. In this section, three methods of parameter constraint were used to model forecasts of the final mean dry head weight. For analysis purposes, the constraints were obtained using full season information. In the next section, actual forecasts were made with and without prior information. The three methods of parameter constraint used for modeling forecasts are numbered one through three for ease of reference. All methods used the model in (1) with the previously described weighting factor. A heteroscedasticity adjustment was not used although further analysis might show that an adjustment would be useful when forecasting mean dry head weights at harvest (i.e. when model is truncated).

There are three parameters that need to be estimated when fitting the basic growth model in (1). Of the three parameters, α is the parameter which we are most interested in estimating since it is the asymptote. As noted earlier, $\hat{\alpha}$ tends to be somewhat larger than the mean dry head weight at harvest and therefore the fitted growth curve should be truncated at the expected date of harvest. Since α, β and ρ are correlated with each other, we need to be aware of their roles in the model to obtain reasonable forecasts of the final $\hat{\alpha}$. If α and β are held constant, increasing values of ρ cause the curve to expand horizontally while keeping the y-intercept the same. This implies that the higher the value of ρ the longer it takes for convergence and, hence, the flatter the curve. To get the characteristic shape of the growth curve in Diagram 1, ρ is constrained to be some value between zero and one. For values of ρ equal to zero and one, the growth model produces horizontal lines at $y = \alpha$ and $y = \alpha/(1+\beta)$, respectively.

If α and ρ are held constant, increasing values of β produce a horizontal shift to the right. The asymptote stays the same while the y-intercept approaches zero as β goes to infinity. This implies that the rate of growth as evidenced by the slope of the tangent at the point of inflection is independent of the value of β. At $\beta = 1$, the point of inflection coincides with the y-intercept. β is constrained to be a positive value so that the curve is in the first quadrant. When $\beta = 0$, a horizontal line is produced at $y = \alpha$.

If β and ρ are held constant, increasing values of α produce a vertical shift up. However, unlike β, the slope of the inflectional tangent increases while the inflection point itself remains at the same value of t. This can be seen since as α increases the y-intercept $(\alpha/(1+\beta))$ increases slower than the asymptote which is α itself. This is true for any values of β and ρ within the previously explained constraints. α is constrained to be a positive value.

Rockwell (1978) suggested that $\hat{\rho}$ be constrained to be within limits obtained from past experience. In method 1, ρ was constrained between .8610 and .8949 which was the range of final values for ρ over all four fields. Parameter estimates and other information are provided in Table A1 of the Appendix for each field at the three forecast dates. The values of $\hat{y}$ are the mean dry head weights at harvest. A compromise value of t = 36 days was used for each field. A comparison of the $\hat{\alpha}$ and $\hat{y}$ values at the forecast dates with the final values as presented in Table 3 shows that there are some large differences. However, except for field 1, the forecasts do tend to approach the final value as more data becomes available. The June 3 forecasts are quite poor in fields 2, 3 and 4. Because of interrelationships among the parameters, the relative standard error (RSE) should be viewed with some discretion when using it for a criteria to assess the reliability of a forecast.

In method 2, in addition to the constraint on ρ, β was constrained between 4.759 and 5.191 which is the range of the final β estimates. Table 3 shows that method 2 produced improvement in the forecasts. Table 4 shows the percent differences between the forecasted $\hat{\alpha}$'s and the final $\hat{\alpha}$'s.

While constraining β and ρ is an improvement over the first method, a pattern has developed in that the final $\hat{\alpha}$ is consistently underestimated on June 3 and June 10 and overestimated on June 17. Furthermore, additional data does not necessarily produce a better forecast in fields 1, 2 and 4. Since β and ρ have both been constrained to be within their final ranges, there may be another factor which should be used in constraining the parameters.

The values of β and ρ determine the point of inflection in the growth curve. This is the point at which the slope of the tangent to the curve is at a maximum. The importance of the point of inflection is that for values of time before the inflection point the growth rate is accelerating and after the inflection point the growth rate slows down. The point of inflection is found by setting the second derivative with respect to t equal to zero and solving for t. When this is done, t is found to be $-(\ln\beta)/(\ln\rho)$. Table 5 shows the points of inflection in days since flowering for each of the forecasts and the final fitted growth curves. There is also a sign with each forecasted inflection point to show the direction that it deviates from the final value.

Table 3.--Modeled forecasts using three methods of parameter constraint

	Forecast dates									
	June 3			June 10			June 17			
Method	1	2	3	1	2	3	1	2	3	Final
Field 1										
Obs.	131	131	131	195	195	195	253	253	253	306
$\hat{\alpha}$	.706	.709	.739	.652	.698	.732	.953	.931	.775	.747
$\hat{y}$ at t=36	.691	.695	.721	.640	.683	.704	.865	.850	.755	.729
Field 2										
Obs.	134	134	134	202	202	202	261	261	261	381
$\hat{\alpha}$	1.727	.879	.993	.805	.826	.947	.988	.988	.964	.960
$\hat{y}$ at t=36	1.457	.859	.956	.789	.808	.894	.929	.929	.915	.908
Field 3										
Obs.	114	114	114	169	169	169	223	223	223	273
$\hat{\alpha}$	1.293	.863	1.003	.789	.903	.944	.972	.960	.954	.948
$\hat{y}$ at t=36	.895	.827	.935	.756	.834	.863	.885	.876	.874	.867
Field 4										
Obs.	109	109	109	167	167	167	222	222	222	326
$\hat{\alpha}$	.404	.851	.848	1.061	.901	.862	.995	.955	.916	.923
$\hat{y}$ at t=36	.402	.783	.780	.941	.823	.801	.899	.872	.854	.859

Table 4.--Percent deviation from final $\hat{\alpha}$
β and ρ constrained to final range (method 2)

	Forecast dates		
Field	June 3	June 10	June 17
1	-5.1	-6.6	+24.6
2	-8.4	-14.0	+2.9
3	-9.0	-4.7	+1.3
4	-7.8	-2.4	+3.5

Table 5.--Points of inflection (method 2)

	Forecast dates			Final
Field	June 3	June 10	June 17	June 24
1	10.4(-)	10.4(-)	14.8(+)	11.0
2	11.0(-)	10.4(-)	13.2(+)	12.7
3	12.5(-)	13.9(-)	14.8(+)	14.7
4	14.0(=)	14.8(+)	14.8(+)	14.0

With the exception of field 4, the direction of the forecasted points of inflection from the final inflection point is the same as the direction of the forecasted $\hat{\alpha}$'s from the final $\hat{\alpha}$ (see Table 4). There is also good agreement in the magnitude of the deviations. Data from previous years were examined to see if the deviation of the forecasted inflection points corresponded to the deviation of the $\hat{\alpha}$ forecasts. The data sets which were examined were 1977 Kansas wheat, 1975 Iowa corn, 1976 Texas corn, 1976 Iowa corn, 1977 Illinois corn and 1977 Iowa corn. Parameter estimates for β and ρ were not available for earlier wheat data sets. In these data sets, the deviation of the inflection points corresponded to the deviation of the $\hat{\alpha}$ forecasts about 95 percent of the time. This apparent relationship might be of use in a forecasting mode if it were possible to know approximately what the final inflection point would be. The corn data sets represent a fairly broad range of situations. The final inflection points are shown in the following table for each of the three models that were used.

Table 6.--Corn points of inflection

Data sets	Models 1	2	3
1975 Iowa	31.3	32.1	30.4
1976 Texas	34.6	32.3	32.4
1976 Iowa	31.9	30.1	30.8
1977 Illinois	31.2	29.8	30.1
1977 Iowa	32.1	30.3	30.9

Except for the 1976 Texas data set which is somewhat different from the others, there is a fairly small range of final inflection points within a particular model. Since the data are from several different states and seasons, this gives some reason to believe that the final inflection point could be used as prior information in a forecasting mode.

In the 1977 wheat data set, the comparable model to the one being used here had a final inflection point of 13.1 which happens to be the mean of the final inflection points in Table 5. In the 1977 Study, it was pointed out (Larsen, 1978) that the final inflection point may be related to the variety. In that study, the inflection points for five major varieties ranged from 10.5 to 15.6. There is not sufficient data to make a conclusion but if a single growth model is fit for wheat in a state, the final inflection point might be fairly consistent over years.

The apparent relationship evidenced in Table 5 suggests that β and ρ be further constrained by their inflection point relationship so that forecasts will have approximately the same inflection point as the final fitted curve. In method 3, β was constrained by the following relationship.

(6) $\beta = \rho^{-IP}$

where IP = the final inflection point in a particular field

Substituting (6) in model (1) gives:

(7) $$y_i = \frac{\alpha}{1 + \rho^{(t-IP)}} + \varepsilon_i$$

The parameter ρ was constrained by its final range. When (7) is fit to the data at the forecast dates, the value of β is completely determined by $\hat{\rho}$ subject to (6). It can be seen that β takes on values in a range somewhat wider than allowed in method 2 but containing the final $\hat{\beta}$ values. Let it be stressed that the $\hat{\beta}$ values in Table A1 were calculated from (6) and not estimated by nonlinear least squares. The following table shows the percent differences between the forecasted $\hat{\alpha}$'s and the final $\hat{\alpha}$'s.

Table 7.--Percent deviation from final $\hat{\alpha}$
ρ constrained to final range, $\beta = \rho^{-IP}$ (method 3)

Field	Forecast dates June 3	June 10	June 17
1	-1.1	-2.0	+3.7
2	+3.4	-1.4	+.4
3	+5.8	-.4	+.6
4	-8.1	-6.6	-.8

A comparison with Table 4 shows that method 3 produces better forecasts in every case except the first two forecast dates in field 4. These two exceptions are the same as those noted in discussing Table 5.

Narrowing the range of the constraint on ρ around the final $\hat{\rho}$ would generally improve the forecasts but not always. The final $\hat{\beta}$ and $\hat{\rho}$ values were substituted into model (1) for each field and only α was estimated by nonlinear least squares. In several cases, the forecasts were actually worse than those in method 3. The improvements tended to be quite small.

Method 3 showed the most potential for forecasting of those considered. For this method to work well in actual practice, the final point of inflection would have to be fairly consistent from year to year. It has been pointed out from other data sets that this may indeed be the case. It would also help in using method 3 if the final $\hat{\rho}$ was fairly consistent from year to year. When Rockwell constrained ρ in the 1977 corn data sets, a very narrow range of .892 to .893 was used based on experience from 1975 to 1976 (Rockwell, 1978). The actual range of the final $\hat{\rho}$ for all the corn data sets listed in Table 6 was .891 to 897. These values were for a growth model which was not adjusted for heteroscedasticity. The adjusted models had somewhat different ranges but still quite narrow. This gives some reason to believe that the final $\hat{\rho}$ in wheat may be fairly consistent from year to year when a single model is fit over many fields.

Forecasting Mean Dry Head Weight

Two variations of the model in (1) were used to provide actual forecasts of the mean dry head weight at harvest. The first forecasting method was done to form a basis of comparison and is referred to as the "control" method. The constraints were as described in (1) with the exception that in field 2 for the June 3 forecast date, β was given an upper limit of 10.0 because otherwise the nonlinear least squares procedure would not have converged.

The second forecasting method used constraints similar to those in method 3. It is referred to as the "reduced" method because fewer parameters are estimated by least squares. Experience from the 1977 wheat data was used to constrain β and ρ. The 1977 final inflection point from a comparable model was 13.1. The estimate of ρ was .8917 with a standard error of .0099. Since there is not a range of $\hat{\rho}$ values from previous years, the constraint on ρ is a range of one standard error on either side of the 1977 $\hat{\rho}$. The constraint is then .8818 to .9016. All the final values of ρ are contained in this interval with the exception of field 1. Table 8 summarizes the forecasts for the two methods. Additional information is contained in Table A2 in the Appendix.

Using prior information to further restrict the range of parameter estimates provided better forecasts of the final $\hat{\alpha}$ and $\hat{y}$ in every case. The improvement is large at the first forecast date. Diagram 2 shows graphs of the forecasted $\hat{\alpha}$'s for the two forecasting methods in each field.

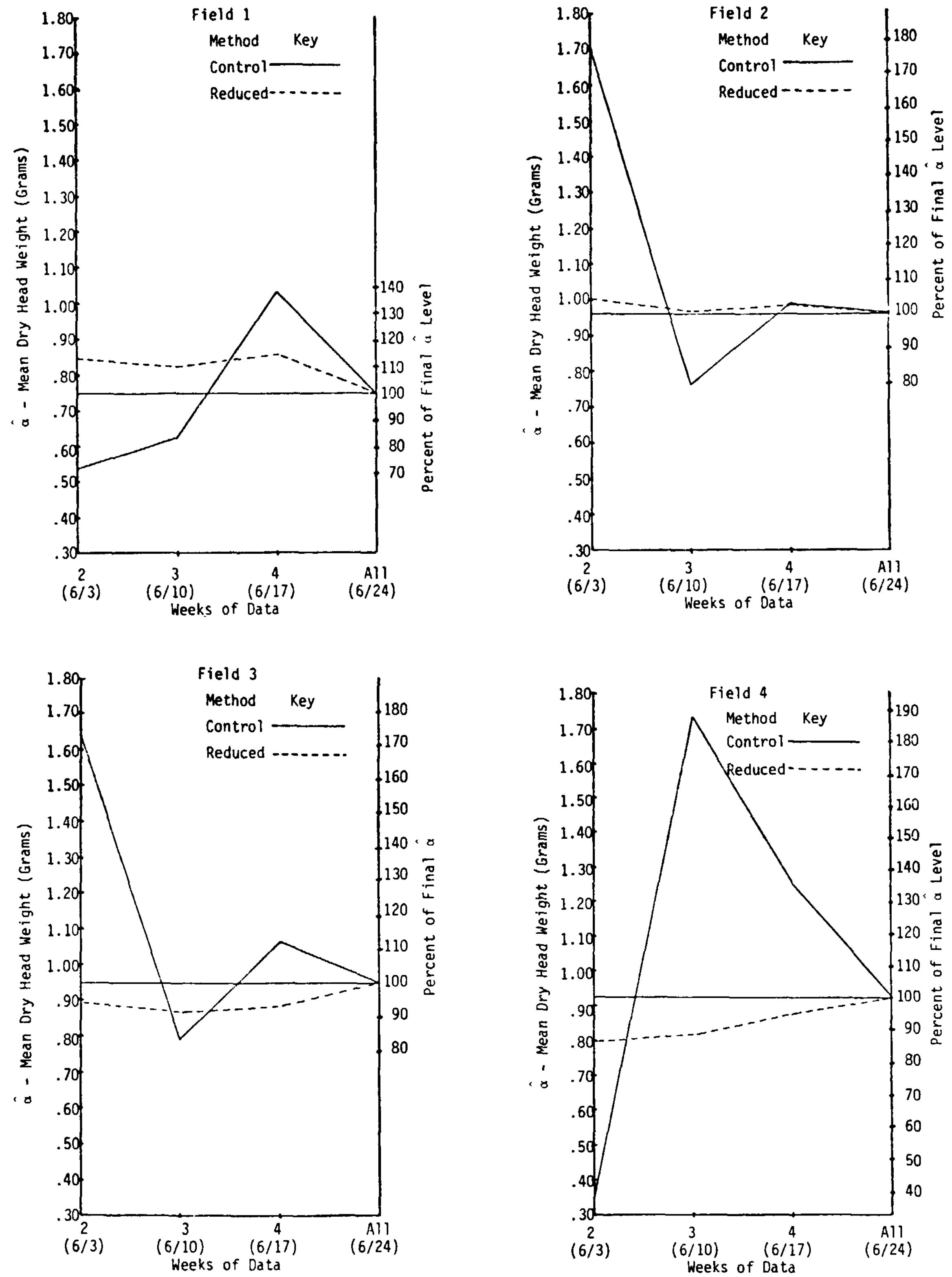
It is important to note that the second method produces forecasts which are high in fields 1 and 2 and low in fields 3 and 4. The point of inflection is always 13.1. Referring back to Table 5, it can be seen that 13.1 is higher than the final inflection point for fields 1 and 2 and lower for fields 3 and 4. As pointed out in the Introduction, the reason for looking at the growth model at the field level was to gain additional insight into the relationships that affect the fitting of a growth model while avoiding some of the problems

Table 8.--Growth model forecasts of final $\hat{\alpha}$ and mean dry head weight at maturity

Method	Forecast dates: June 3 Control	June 3 Reduced	June 10 Control	June 10 Reduced	June 17 Control	June 17 Reduced	Final	Calculated dry mean head weight
Field 1								
Obs.	131	131	195	195	253	253	306	
$\hat{\alpha}$	.537	.845	.621	.822	1.032	.856	.747	
$\hat{y}$ at t=36	.536	.800	.614	.764	.897	.811	.729	.687
Field 2								
Obs.	134	134	202	202	261	261	381	
$\hat{\alpha}$	1.707	1.001	.763	.968	.988	.983	.960	
$\hat{y}$ at t=36	1.445	.948	.754	.907	.929	.927	.908	.891
Field 3								
Obs.	114	114	169	169	223	223	273	
$\hat{\alpha}$	1.634	.895	.789	.867	1.062	.883	.948	
$\hat{y}$ at t=36	1.318	.848	.756	.809	.923	.835	.867	.783
Field 4								
Obs.	109	109	167	167	222	222	326	
$\hat{\alpha}$	.343	.787	1.734	.819	1.255	.877	.923	
$\hat{y}$ at t=36	.343	.720	1.163	.771	.990	.830	.859	.891
Mean $\hat{y}$	.911	.829	.822	.813	.935	.851	.841	.813

associated with fitting a single model over many fields. While making field level growth model forecasts in many fields and aggregating to the state level is conceivable, the real application of the growth model in an operational program is fitting a single model (or possibly several models by variety). When data from many fields are used in a growth model, it is reasonable to believe that the inflection point over years might remain fairly consistent (as already pointed out in the corn data) even though the inflection points in individual fields might differ. Since the inflection point in the second forecasting method also happens to be the mean of the four final inflection points, this method should do a good job of forecasting the mean $\hat{\alpha}$ or the mean $\hat{y}$ over all fields. If the model forecasts are averaged, the forecasts of either the mean

Diagram 2 -- Growth model forecasts of final $\hat{\alpha}$



$\hat{\alpha}$ or $\hat{y}$ are within 4 percent on the first two forecast dates and within 2 percent on the third forecast date. The forecasts also compare well with the calculated mean dry head weight. While being able to forecast a mean $\hat{\alpha}$ or $\hat{y}$ for only four fields is not particularly meaningful as far as inference is concerned, it does suggest that large area forecasts might be possible by either aggregating many field level forecasts using a common inflection point or by fitting a single model to data from many fields.

Relationship Between Flowering Date and Mean Dry Head Weight

The data from field 3 and the corresponding fitted growth curve are shown in Figure 1 in the Appendix. The horizontal time variable is days after the initial observance of flowering. The vertical axis is dry head weight in grams. The data appears to be grouped into 7-day intervals of time each containing observations for three values of time. Dotted lines have been drawn to show this grouping which corresponds to the weekly clip dates and thrice weekly visits to observe flowering and define maturity stages. When moving from one 7-day interval to the next, there is a distinct decrease in the relative magnitude of the dry head weights. The method of data collection has induced the appearance of grouping in the data which likely would not have been as visible if a random sample of heads had been taken on fixed calendar dates without regard to time since flowering. However, it is legitimate to conclude that there are at least three different growth relationships within the field. Plots of the data from the other fields revealed similar shifts when moving from one 7-day interval to another. The different growth relationships within a field contribute to the magnitude of the relative standard errors.

The different growth relationships within fields do not directly correspond to the maturity stages as they have been defined. The reason is that the maturity stages were defined on a plot basis so the first maturity stage in one plot might have a different flowering date than the first maturity stage in another plot. A set of plots were made where the data points were represented by symbols indicating the flowering date. Figure 2 shows the plot for field 3. The letters range from A to P corresponding to julian flowering dates ranging from 139 to 154. It is evident from Figure 2 that the different growth relationships are dependent at least in part on the flowering date. This is important because the growth model utilizes the relationship between time since flowering and head weight but does not directly address the relationship between flowering date and head weight. To do this, the data should be divided into groups with common or nearly common flowering dates and growth models fit individually to the groups within each field. The maturity stages do this but the meaning of the maturity stage numbers needs to be consistent over all plots within a field. After examining the data from all four fields, it was determined that flowering dates could be grouped into four categories with sufficient observations in each for analysis. These new categories were called flowering maturity stages (FMS) and the definition of each is as follows:

FMS	Julian Flowering Date (JFD)
1	$JFD \leq 141$
2	$141 < JFD \leq 143$
3	$143 < JFD \leq 145$
4	$JFD > 145$

Figure 3 shows a plot of the field 3 data with observations represented by FMS numbers. The FMS categories appear to correspond to the different growth relationships.

To further investigate the relationship between different flowering maturity stages, unadjusted growth models were fit for each FMS within a field. Table 9 contains some of the information. Significant correlations indicated that a heteroscedasticity adjustment is needed for most of the fits but the adjusted model was not used because it was seen earlier that heteroscedasticity has little affect on the mean dry head weight at harvest when the model is truncated. It can be seen that $\hat{y}$ tends to be higher for earlier flowering heads. This tendency is not so evident for $\hat{\alpha}$ partially because the later maturity stages have very imprecise estimates. The differences between the flowering maturity stages within fields is quite large in some cases. It is possible to weight the four mean dry head weights within a field together to obtain one mean per field. The weights which are needed to do this are presented in a later section.

Estimation of Final Harvested Yield

The final harvested yield can be estimated from the dry kernel weight data collected on the final pre-harvest visit. As described earlier, there are two data sets with dry kernel weights. For the dry kernel weights obtained on the regularly sampled heads, the grain weight per square foot can be calculated as follows:

$$(8) \quad WTPSQFT = \sum_i \sum_j \frac{(FSPSQFT_i \ \ FSPFMS_{ij} \ \ DKW_{ij})}{24 \ \sum_j FSPFMS_{ij}}$$

where:
WTPSQFT = grain weight per square foot
FSPSQFT = number of flowered stalks per square foot
FSPFMS = number of tagged stalks which flowered in each flowering maturity stage
DKW = dry kernel weight per head
$i = 1, 2, \ldots, 24$ plots
$j = 1, 2, 3, 4$ flowering maturity stages

Table 9.--Growth model fit at FMS level

Field	FMS	Obs.	MSE	R^2	$\hat{\alpha}$ (g)	RSE (%)	$\hat{y}$ at (g)	t (days)
1	1	34	.009	.973	.706	6.7	.681	36
	2	105	.024	.953	.743	5.1	.714	33
	3	105	.031	.932	.759	5.6	.734	31
	4	52	.012	.846	1.486	339.6	.465	29
2	1	33	.017	.977	.989	4.8	.970	36
	2	131	.037	.936	.971	6.4	.893	33
	3	137	.046	.909	.794	4.6	.775	31
	4	80	.075	.846	.843	10.4	.798	29
3	1	14	.008	.991	.974	9.8	.910	35
	2	116	.025	.940	.952	9.9	.835	32
	3	106	.018	.923	.654	9.2	.599	30
	4	37	.016	.842	.546	62.3	.436	28
4	1	8	.010	.989	.781	15.6	.717	37
	2	136	.049	.934	.868	6.8	.808	34
	3	113	.058	.895	1.532	44.2	.978	32
	4	69	.047	---	1.168	---	.823	30

The mean grain weight per square foot was converted to bushels per acre and adjusted from near zero percent moisture to the standard 12 percent. The mean harvest loss calculated from eight plots per field was subtracted to give a net yield per acre. Data were collected at the elevator to form a basis of comparison. All the elevator yields are very good since normal yields tend to be around 30 bushels per acre in Ellsworth County, Kansas. The estimated yields are all high ranging from a 27 percent overage in field 1 to 81 percent in field 4. The fact that all four fields are high is possible with random errors but a bias is suspected.

The net yield can also be calculated from the extra clipped heads. Since these heads were not from the tagged stalks, it is only necessary to weight the plot mean grain weight by the mean number of flowered stalks at harvest. The resultant field level yields compared well with those from the regular sample presented in Table 10 (see Table A3 in the Appendix).

There are several things that may be causing an upward bias in the net yield estimates. The harvest loss units tend to consistently indicate average losses of two to three bushels and are thought to adequately measure the grain loss within the field. However, it is suspected that at least as much or more loss may occur while loading trucks along the edge of the field or while the grain is in transit to the elevator. These kinds of losses are difficult to measure and, hence, one can only speculate as to their magnitude.

Table 10.--Comparison of estimated yield and elevator yield

Field	Obs.	Grain wt. at 12% (bu/ac)	Harvest loss (bu/ac)	Net est. yield (bu/ac)	Net elev. yield (bu/ac)
1	52	58.6	2.4	56.2	44.2
2	59	101.7	2.1	99.6	62.4
3	50	70.1	2.6	67.5	41.8
4	51	81.3	3.0	78.3	43.3

Another of the components in the yield estimate which should come under scrutiny is the estimate of stalk population. Based on the 1977 wheat data, it was anticipated that 1978 stalk counts within the 5-foot plots would be in the neighborhood of 200. However, stalk counts of 400 and 500 were very common. When the counts were made in the spring of 1978, denser stands could be explained by the favorable winter and spring weather which had encouraged tillering. The percentage of stalks which eventually flowered and survived until harvest was much lower than found in 1977. This caused the estimated flowered stalk populations at maturity to be closer to the 1977 results but still much higher. As would be expected, the denser stands did cause the mean dry head and kernel weights to be less than those estimated in the 1977 Study. It is felt that the method used to estimate the proportion of stalks which flowered and survived to harvest is sound. The early spring stalk counts and procedures were reviewed but no errors or inherent bias could be identified.

The relationship between the flowering date and head weight, mentioned earlier, suggested a closer examination of the procedure for defining maturity stages and sampling stalks to see if this could be causing any problems in the yield estimates. With the possible exception of the last maturity stage, all maturity stages contained at least seven stalks. In cases where maturity stages included less than seven stalks, sampling continued until all stalks had been clipped leaving no stalks to be clipped as harvest approached. It was originally thought that this would have little affect on the estimates. Based on the 1977 Study, it was also believed that flowering would be spread out enough to allow the definition of approximately seven maturity stages per plot. Since generally only two or three were identified, the proportion of "last" maturity stages to total maturity stages was much higher than expected.

The "last" maturity stages roughly correspond to the last flowering maturity stages (FMS) which were described earlier. Since the head weights tend to be lower for later flowering stalks, the fact that the later flowering maturity stages were not represented in the sample near harvest may be causing an upward bias. Additionally, since the flowering maturity stages were defined consistently over all plots, there are a few more flowering maturity stages than there were original maturity stages. This means that more flowering maturity stages are lacking observations on the final pre-harvest visit than were the original maturity stages.

It is difficult to assess the affect of the missing observations from final pre-harvest visits when dry kernel weight was obtained. The affect is somewhat offset by weighting the dry kernel weights by the number of flowered stalks within FMS (see equation (8)). The weight is low for the FMS which did not get sampled on the final pre-harvest visit because the number of stalks in the FMS was less than seven. Nevertheless, not having the smaller heads represented adequately in the yield estimates would tend to bias them upward. A method for assessing the affect of the bias was developed.

Estimation of Missing Values

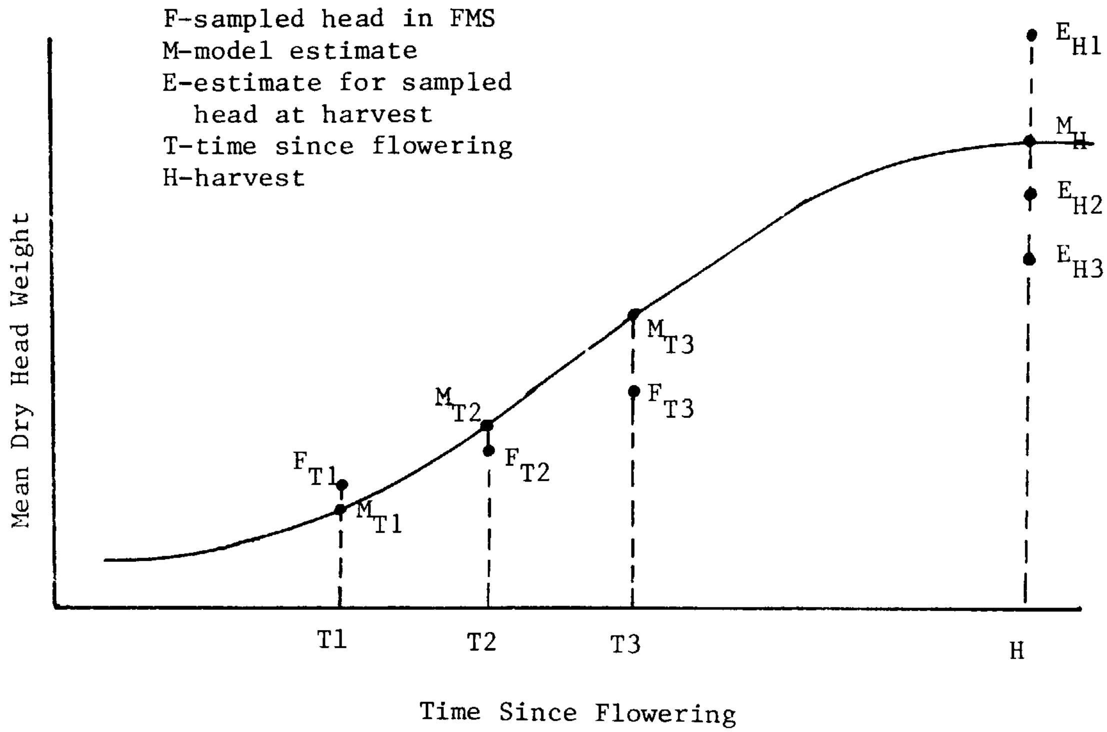
The missing values can be estimated by using the fitted growth curves in Table 9. Each FMS with a missing value on the final pre-harvest visit has at least one stalk which was sampled earlier in the season. The fitted growth curves can be used to forecast the weight of the heads from each of these stalks had they continued to grow until harvest. This is done by using the time since flowering for each of the head weights present in the FMS in the appropriate fitted curve to estimate the mean dry head weight at the same point in time. The ratio of the actual head weight divided by the model estimated head weight at the same point in time is obtained. This ratio multiplied times the estimated mean dry head weight at harvest ($\hat{y}$) provides an estimate of what the sampled head would have produced at harvest (see Diagram 3). Using this procedure for each stalk in the FMS and averaging the forecasts at harvest, provides an estimate for the missing value. There were a total of 79 missing values which were estimated in this manner.

Table 11 summarizes the yield estimates when the missing value estimates are supplied. Comparing with Table 10, it can be seen that the affect is very small. This is primarily due to the low weights applied to the "last" maturity stages. This result seems to suggest that even though the sampling design did not represent the late maturity stages adequately, the bias is inconsequential because of the low number of stalks in the late stages. It is gratifying to find that the sampling design does not appear to be deficient, at least for this data set, but the reason for the high yield estimates remains unexplained.

Table 11.--Estimated yield using missing value estimates

Field	Obs.	Grain wt. at 12% (bu/ac)	Harvest loss (bu/ac)	Net est. yield (bu/ac)
1	75	58.4	2.4	56.0
2	81	101.0	2.1	98.9
3	68	69.3	2.6	66.7
4	67	80.4	3.0	77.4

Diagram 3
Method Used to Estimate
Missing Values



$$E_{H1} = \frac{F_{T1}}{M_{T1}} \cdot M_H$$

$$E_{H2} = \frac{F_{T2}}{M_{T2}} \cdot M_H$$

$$E_{H3} = \frac{F_{T3}}{M_{T3}} \cdot M_H$$

$$\text{Estimate of Missing Value} = \frac{E_{H1} + E_{H2} + E_{H3}}{3}$$

Model Estimated DHW and DKW/DHW Ratio at FMS Level

In practice, the mean dry head weights at harvest from the fitted growth models (Table 9) would be adjusted to mean dry kernel weight using the relationship of head weight and kernel weight from heads sampled just before harvest. These mean dry kernel weights can then be weighted together to obtain a growth model estimate of harvested yield. The relationship between head weight and flowering date discussed earlier is cause to investigate the use of the mean dry head weights from growth models fit by maturity stage. Weighting these means together to obtain a field level mean weight per unit area is somewhat of a problem. The weights can be developed by calculating the mean dry kernel weights by maturity stage and seeing what weights are needed to obtain the previously calculated field level weight per square foot in (8). The formula for calculating mean dry kernel weight by maturity stage is as follows. The WTPSQFT in (10) is equivalent to (8). The X_j serves as a weighting factor. Table 12 shows the

$$(9)\quad MEANDKW_j = \frac{\sum_i (FSPSQFT_i \; FSPFMS_{ij} \; DKW_{ij})}{\sum_i (FSPSQFT_i \; FSPFMS_{ij})}$$

$$(10)\quad WTPSQFT = \sum_j X_j \; MEANDKW_j$$

$$\text{where } X_j = \sum_i \frac{FSPSQFT_i \; FSPFMS_{ij}}{24 \sum_j FSPFMS_{ij}}$$

comparison between the mean dry kernel weight measured directly (MEANDKW) and the mean dry kernel weight derived from the growth model estimated mean dry head weight and kernel weight to head weight ratio (MEANDKW2). The MEANDHW in Table 12 is from Table 9 ($\hat{y}$ at t). As was the case with the head weights, it can be seen that the kernel weights tend to decrease as the maturity stage increases. It is interesting to note that the kernel weight to head weight ratio has a similar relationship to the maturity stage. It can be seen that MEANDKW2 and MEANDKW compare quite well in most cases. This suggests that the growth model is generally doing a good job of estimating the mean dry head weight present at harvest.

Table 13 shows the calculated yield estimates using the growth model estimates in Table 12 and equation (10). Comparing the growth model calculated yield with the yield calculated directly from the kernel weight in Table 10 reveals at most a four bushel difference or about 5 percent. A paired t-test with 3 degrees of freedom shows a significant different between the two means at α = .70.

Table 12.--Comparison of mean DKW calculated directly and from model estimated mean DHW and DKW/DHW ratio

Field	FMS	MEANDKW (g)	X	MEANDHW (g)	DKW/DHW ratio	MEANDKW2 (g)
1	1	.511	4.91	.681	.741	.505
	2	.509	34.37	.714	.736	.526
	3	.469	24.94	.734	.697	.512
	4	.285	1.68	.465	.617	.287
2	1	.697	2.27	.970	.774	.751
	2	.670	62.80	.893	.713	.637
	3	.565	16.83	.775	.733	.568
	4	.598	4.59	.798	.709	.566
3	1	.646	1.10	.910	.737	.671
	2	.582	56.98	.835	.696	.581
	3	.380	11.97	.599	.704	.422
	4	.166	.69	.436	.480	.209
4	1	.530	1.03	.717	.707	.507
	2	.664	41.98	.808	.752	.608
	3	.681	20.61	.978	.730	.714
	4	.590	3.30	.823	.693	.570

Table 13.--Estimated yield using model estimated DHW and DKW/DHW ratio at FMS level

Field	Grain wt. at 12% (bu/ac)	Harvest loss (bu/ac)	Net est. yield (bu/ac)
1	61.5	2.4	59.1
2	98.0	2.1	95.9
3	71.0	2.6	68.4
4	77.6	3.0	74.6

Model Estimated DHW and DKW/DHW Ratio at Field Level

Even though there is a relationship between head weight and flowering date, it may be possible to do about as well using a single fitted growth curve in a field because of the weighting that was used. To investigate this, the yield was calculated using field level means. Table 14 shows the components which were used in the calculation. The MEANDHW is the unadjusted mean dry head weight from Table 2. The yield was calculated from the product of the MEANDHW, DKW/DHW ratio and the mean flowered stalks per square foot. The estimated yields compare fairly well with those in Table 13 but are all higher. A paired t-test with 3 degrees of freedom shows a significant difference at $\alpha = .10$. A comparison with Table 10 where the actual kernel weight was used reveals good agreement with the possible exception of field 3 where the fallow and no fallow areas may be causing the growth model estimate of mean dry head weight to increase. A paired t-test with 3 degrees of freedom shows a significant difference at $\alpha = .40$.

Table 14.--Estimated yield using model estimated DHW and DKW/DHW ratio at field level

Field	MEANDHW (g)	DKW/DHW ratio	FSPSQFT	Net est. yield (bu/ac)
1	.729	.718	65.90	60.3
2	.908	.718	86.49	100.5
3	.858	.696	70.73	74.2
4	.866	.742	66.93	75.2

Reliability of DKW/DHW Ratio Using Extra Heads

As discussed earlier, the main purpose for clipping extra heads which are adjacent to the regular sample on the final pre-harvest visit is to obtain a DKW/DHW ratio. As pointed out, data collected for the 1977 Study showed such high variability in this ratio that its use was questioned. This problem can now be directly addressed since dry kernel weight data were collected on the regular heads as well as the extra heads. The DKW/DHW ratio was calculated using the extra heads on the maturity stage level for possible use as in Table 12. These alternative ratios bore very little resemblance to those in Table 12. Several exceeded 1.0 in the later maturity stages due to the fact that the pairing did not create a strong enough relationship for the extra heads to correspond in maturity. If adjacent pairs of heads do not tend to fall into the same maturity stage, the sample of extra heads is really more like a random sample and the ratio obtained by using extra heads could not be of any use when calculated by maturity stages. With a sufficiently large sample, however, field level mean DKW/DHW ratios using extra heads may correspond well enough to the ratios in Table 14 to be of use. Table 15 summarizes the two ratios at the field level. With approximately 50 observations in the field, the correspondence is not very good. The ratio from the extra heads ranges from 10 to 35 percent higher. A paired t-test, however, only indicates a significant difference between the means at $\alpha = .06$. An explanation for why the extra head ratio is higher in every case is not obvious particularly since the mean dry kernel weights from the extra heads are not, in general, higher than the corresponding means from the regular heads (see Table A3). Obtaining the DKW/DHW ratio from the regular heads is preferable to clipping adjacent heads.

Table 15.--Comparison of two DKW/DHW ratios

	Regular heads only		Extra and regular heads	
Field	DKW/DHW ratio	Obs.	DKW/DHW ratio	Obs.
1	.718	52	.789	48
2	.718	59	.967	57
3	.696	50	.783	46
4	.742	51	.829	49

Summary and Conclusion

The 1978 Study answered several questions while raising others. In summary, some of the things which were learned from the effort are as follows. The most encouraging thing to come out of the study is that it was possible to make June 1 field level forecasts of mean dry head weight at harvest. This was accomplished by constraining the parameters to be estimated in the growth model using information from the previous year. When the parameter estimates were constrained to be within a range of values consistent with past experience, the early forecasts were greatly improved. The June 1 forecasts were approximately 15 percent away from the final values in two fields and about 5 percent away in the other two fields. However, a relationship between the direction of the missed forecast and the point of inflection was identified. As a result of this relationship, it is possible that a large area forecast using either the aggregate of many field level forecasts or, preferably, a single model fit to many fields would be within 5 percent of the final level on June 1. This accuracy was demonstrated with the 1978 data by aggregating the four fields.

The importance of sampling on the basis of maturity stages so that aggregation could take place without averaging over differing time values was indicated in the 1977 analysis. This was not so important in 1978 because of a shorter flowering period. The 1978 Study revealed the relationship of the flowering date and, hence, the maturity stage to the head weight and other yield components. The existence of this relationship was not previously recognized.

Another concept which was used for the first time in this type of research was truncation of the fitted growth model near the harvest date. The asymptote α is the mean dry head weight when the time variable is large. However, the estimated mean dry head weight at harvest, the quantity of actual interest, was somewhat less. It was also seen that in the truncated model, a heteroscedasticity adjustment made little difference in the estimate of the mean dry head weight at harvest. Whether this is also the case for forecasts was not evaluated. In practice, the harvest date could be forecasted by using the fairly consistent length of time between flowering and harvest.

Obtaining dry kernel weight on both regular heads and extra heads revealed that unless a very large sample is used, the pairing of the extra heads is not strong enough to provide reliable kernel to head weight ratios. The sample design was generally adequate. Any bias caused by not representing the later maturity stages properly in the sample appeared to be nearly offset by the weighting. A method for estimating missing values was developed to evaluate the affect of the bias. The sample design could be improved by defining maturity stages on a field level rather than the plot level. The number of plots and heads sampled within plots appeared to be adequate. If standard deviations of the mean are calculated rather than those in Table A3, the resultant coefficients of variation would be 5 percent or less for all the different counts and weights which were estimated.

The main question which was raised by the 1978 Study and remains unanswered is why the yield estimates were so much higher than the actual yield measured at the elevator. Several possible sources of bias were put forth. When it was discovered that the late maturity stages tend to yield less than the earlier ones and the sample design did not represent the late maturity stages adequately, it was thought that this might be a major contributor to the high estimates. All missing values were estimated and it was shown that since the late maturity stages received much less weight than the early ones, the missing values really didn't have much of an impact on the aggregate yield estimates. Another possible source of bias was the harvest loss estimates which were likely to be somewhat low due to the inability to measure any losses outside of the field. This factor, however, would only account for a small portion of the overestimates. The possible source of bias which remains in question is the initial stalk counts. While much higher than encountered in the 1977 Study, the lower percentage of flowered stalks and the lower mean head weight made somewhat higher initial stalk counts believable. The procedure used to count stalks and estimate the percent which flowered appeared to be accurate so it was not determined why the estimate of flowered stalks present at maturity would be biased upward. If a count of the total number of heads in the plot on the final pre-harvest visit had been made, the accuracy of the estimated number of heads at harvest could have been evaluated.

In conclusion, it was possible to forecast and estimate the mean dry head weight at harvest using the growth model. It was also possible to estimate the mean grain weight per head using dry kernel weight to dry head weight ratios obtained from heads sampled just prior to harvest. To be able to forecast the mean grain weight per head, a dry kernel weight to dry head weight ratio from previous years would have to be applied. The range of ratios in the four fields was about 6 percent. The mean ratio over many fields is believed to be quite consistent over years but this has not been assessed. To obtain biological yield, an estimate of the number of heads per unit area at harvest was calculated. This component, which was based on an April stalk count, is thought to be high. The number of heads per unit area could alternatively be forecasted and estimated by counting all the heads present in the plot during or shortly after flowering and again just before harvest. The harvested yield was estimated by subtracting harvest loss from the biological yield and was found to be higher than the yield measured at the elevator for all four fields.

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Appendix

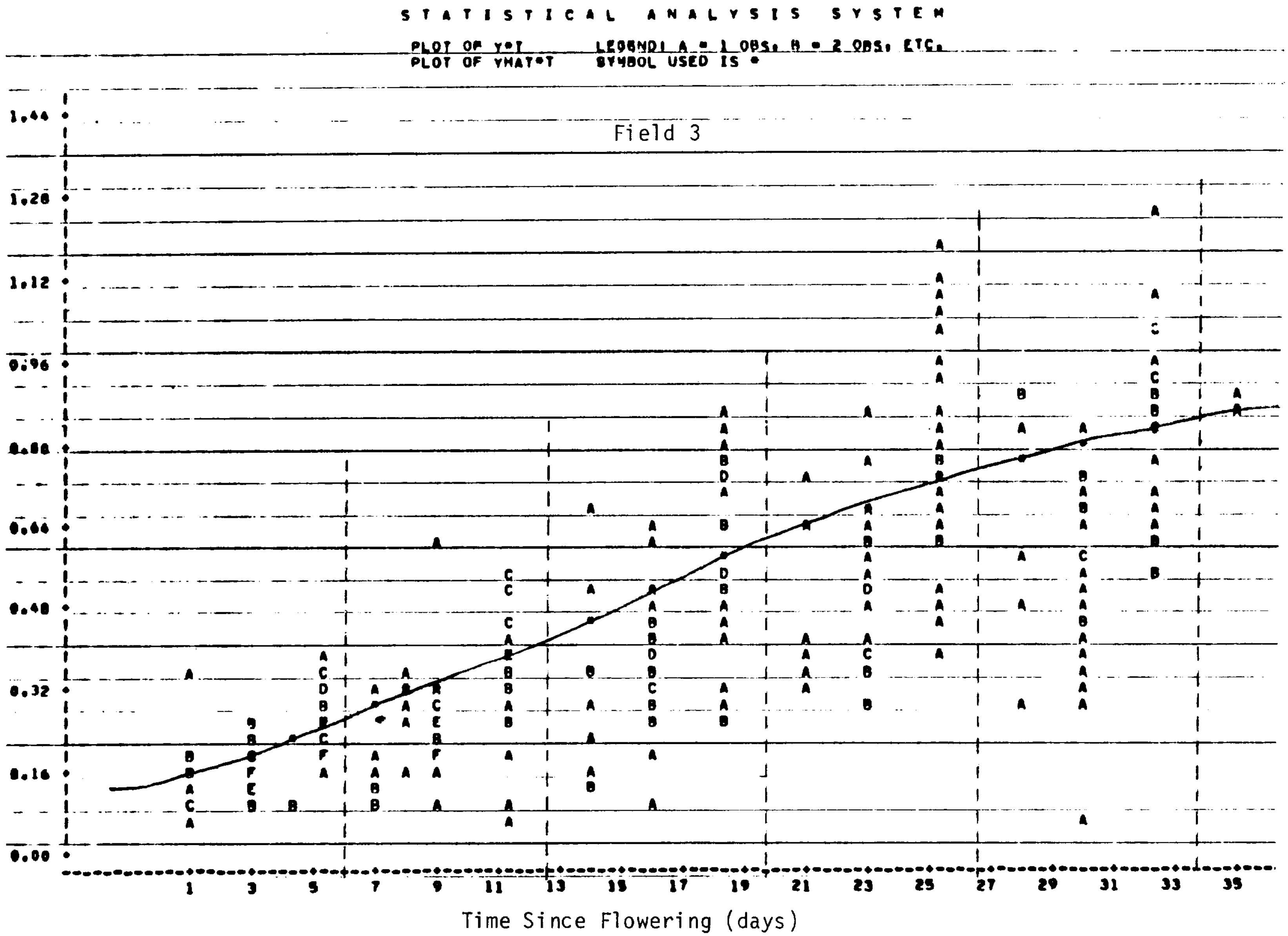
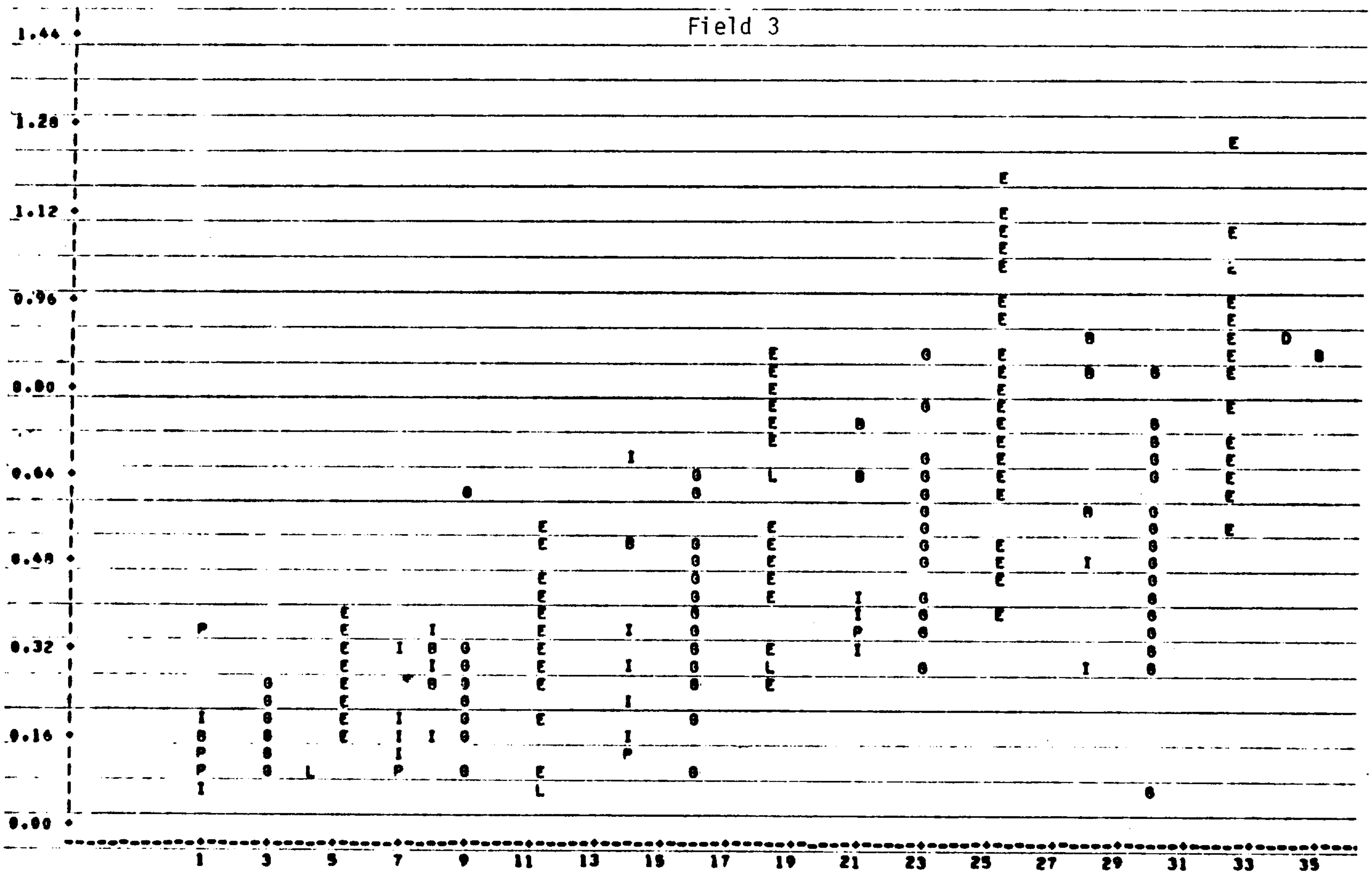

S T A T I S T I C A L   A N A L Y S I S   S Y S T E M
PLOT OF Y*T   LEGEND: A = 1 OBS, B = 2 OBS, ETC.
PLOT OF YHAT*T   SYMBOL USED IS *
Field 3
Dried Head Weight (grams)
1.44
1.28
1.12
0.96
0.80
0.64
0.48
0.32
0.16
0.00
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
33
35
Time Since Flowering (days)

Figure 1

S T A T I S T I C A L   A N A L Y S I S   S Y S T E M

PLOT OF DHW*FTIME   SYMBOL IS VALUE OF Q



Dried Head Weight (grams)

Time Since Flowering (days)

Figure 2

STATISTICAL ANALYSIS SYSTEM

PLOT OF DHW*FTIME SYMBOL IS VALUE OF FMS

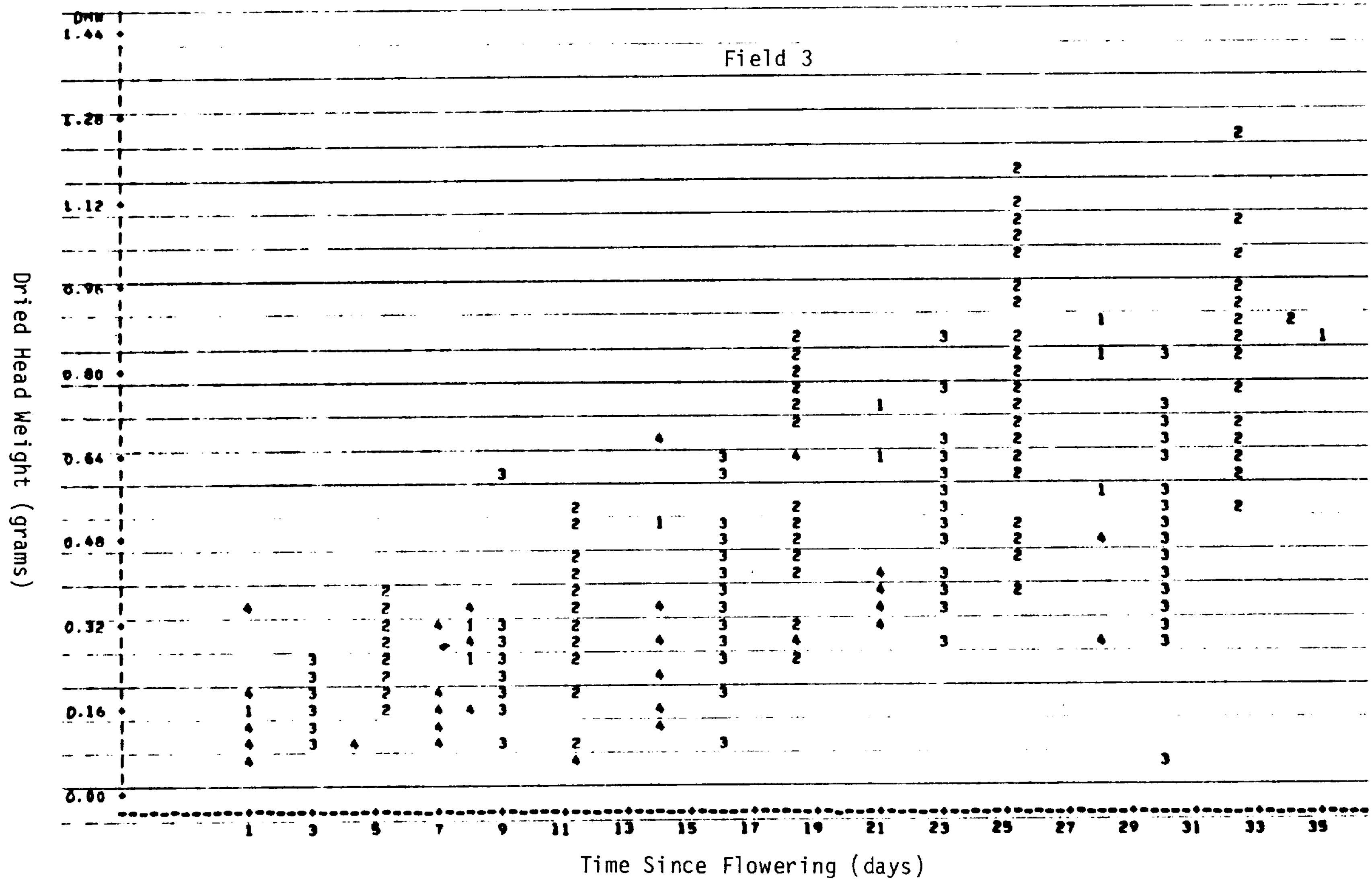


Figure 3

Table A1.--Modeled forecasts using three methods of parameter constraint

	Forecast dates									
	June 3			June 10			June 17			
Method	1	2	3	1	2	3	1	2	3	Final
Field 1										
Obs.	131	131	131	195	195	195	253	253	253	306
R^2	.962	.962	.962	.962	.961	.961	.950	.950	.947	.944
$\hat{\alpha}$	.706	.709	.739	.652	.698	.732	.953	.931	.775	.747
RSE	48.5	48.8	1.8	8.8	9.6	1.6	10.6	10.2	2.0	3.2
$\hat{\beta}$	4.716	4.759	4.976	3.970	4.759	4.139	5.557	5.191	4.990	5.178
$\hat{\rho}$	.8610	.8610	.8643	.8610	.8610	.8789	.8949	.8949	.8641	.8610
$\hat{y}$ at t=36	.691	.695	.721	.640	.683	.704	.865	.850	.755	.729
Field 2										
Obs.	134	134	134	202	202	202	261	261	261	381
R^2	.954	.954	.954	.933	.933	.932	.930	.930	.930	.933
$\hat{\alpha}$	1.727	.879	.993	.805	.826	.947	.988	.988	.964	.960
RSE	354.7	94.9	2.3	14.6	13.4	2.0	9.4	9.4	2.1	3.8
$\hat{\beta}$	10.11	5.191	5.868	4.368	4.759	4.650	4.934	4.934	4.922	4.759
$\hat{\rho}$	.8949	.8610	.8699	.8610	.8610	.8860	.8860	.8860	.8821	.8843
$\hat{y}$ at t=36	1.457	.859	.956	.789	.808	.894	.929	.929	.915	.908
Field 3										
Obs.	114	114	114	169	169	169	223	223	223	273
R^2	.945	.945	.945	.935	.935	.935	.925	.925	.925	.937
$\hat{\alpha}$	1.293	.863	1.003	.789	.903	.944	.972	.960	.954	.948
RSE	545.1	250.6	4.1	24.6	37.5	2.1	15.3	15.1	2.1	6.8
$\hat{\beta}$	7.866	5.191	6.051	4.293	4.759	5.116	5.364	5.191	5.225	5.113
$\hat{\rho}$	.8949	.8762	.8847	.8800	.8937	.8949	.8949	.8949	.8936	.8949
$\hat{y}$ at t=36	.895	.827	.935	.756	.834	.863	.885	.876	.874	.867
Field 4										
Obs.	109	109	109	167	167	167	222	222	222	326
R^2	.906	.893	.893	.906	.905	.905	.919	.919	.919	.916
$\hat{\alpha}$	.404	.851	.848	1.061	.901	.862	.995	.955	.916	.923
RSE	48.0	249.7	5.3	56.7	46.2	2.6	16.7	15.7	2.2	5.7
$\hat{\beta}$	1.054	4.759	4.733	6.972	5.191	5.136	5.790	5.191	5.332	5.191
$\hat{\rho}$	.8610	.8949	.8949	.8949	.8949	.8897	.8949	.8949	.8873	.8889
$\hat{y}$ at t=36	.402	.783	.780	.941	.823	.801	.899	.872	.854	.859

Table A2.--Growth model forecasts for control and reduced methods

	Forecast dates						
	June 3		June 10		June 17		
Method	Control	Reduced	Control	Reduced	Control	Reduced	Final
Field 1							
Obs.	131	131	195	195	253	253	306
R^2	.963	.962	.962	.961	.950	.949	.944
$\hat{\alpha}$	.537	.845	.621	.822	1.032	.856	.747
RSE	19.8	2.2	7.1	1.5	14.0	1.8	3.2
$\hat{\beta}$	4.056	5.196	3.999	4.366	5.629	5.196	5.178
$\hat{\rho}$	.8156	.8818	.8478	.8936	.9043	.8818	.8610
$\hat{y}$ at t=36	.536	.800	.614	.764	.897	.811	.729
Field 2							
Obs.	134	134	202	202	261	261	381
R^2	.954	.954	.933	.932	.931	.930	.933
$\hat{\alpha}$	1.707	1.001	.763	.968	.988	.983	.960
RSE	348.8	2.4	9.8	2.0	9.4	2.0	3.8
$\hat{\beta}$	10.00	5.196	4.519	4.699	4.934	4.932	4.759
$\hat{\rho}$	.8946	.8818	.8468	.8886	.8860	.8853	.8843
$\hat{y}$ at t=36	1.445	.948	.754	.907	.929	.927	.908
Field 3							
Obs.	114	114	169	169	223	223	273
R^2	.945	.945	.935	.935	.925	.924	.937
$\hat{\alpha}$	1.634	.895	.789	.867	1.062	.883	.948
RSE	794.1	3.3	24.6	2.0	20.7	2.2	6.8
$\hat{\beta}$	10.00	5.196	4.293	4.544	5.489	5.098	5.113
$\hat{\rho}$	.9015	.8818	.8800	.8909	.9050	.8831	.8949
$\hat{y}$ at t=36	1.318	.848	.756	.809	.923	.835	.867
Field 4							
Obs.	109	109	167	167	222	222	326
R^2	.908	.898	.908	.904	.920	.918	.916
$\hat{\alpha}$	.343	.787	1.734	.819	1.255	.877	.923
RSE	6.3	4.7	158.9	2.5	33.6	2.3	5.7
$\hat{\beta}$	1.717	3.884	10.00	4.866	6.375	5.196	5.191
$\hat{\rho}$	.6755	.9016	.9197	.8862	.9158	.8818	.8889
$\hat{y}$ at t=36	.343	.720	1.163	.771	.990	.830	.859

Additional Summary Statistics

Other types of data were collected during the 1978 Study. Means and standard deviations for these additional types of data, along with some other statistics which have not yet been presented, are contained in Table A3. The standard deviations for the regular clipped heads on dry kernel weight per head (DKW), number of fertile spikelets per head (FS) and number of kernels per head (KC) are thought to be a maximum estimate of the true population standard deviations. The reason for this is that weighted variance formulas had to be used which tend to form an upper limit for the true variations. The standard deviations for the extra clipped heads were calculated using the usual formulas and are noticeably smaller. Most likely, the major reason for this is the use of the differing variance formulas. The means from the two sets of heads compare quite well in most cases with differences going in both directions. Paired t-tests indicate no significant differences.

Table A3.--Additional summary statistics

Regular clipped heads

Field	Obs.	DKW Mean (g)	DKW S.D. (g)	FS Mean	FS S.D.	KC Mean	KC S.D.	FSPSQFT Mean	FSPSQFT S.D.
1	52	.488	.055	10.66	.685	18.08	1.391	65.90	12.93
2	59	.649	.064	13.40	.576	24.97	1.888	86.49	20.33
3	50	.544	.061	11.07	.750	18.65	1.539	70.73	25.36
4	51	.665	.072	11.92	.705	20.64	1.473	66.93	20.15

Extra clipped heads

Field	Obs.	DKW Mean (g)	DKW S.D. (g)	FS Mean	FS S.D.	KC Mean	KC S.D.
1	48	.461	.027	10.24	.452	17.63	.800
2	57	.689	.027	14.28	.215	27.29	.820
3	46	.514	.033	10.50	.493	18.61	1.036
4	49	.663	.041	11.82	.544	21.67	.914